

Mathematical Analysis - List 13

1. Find the average value of $f(x) = 5x^2\sqrt{1+x^3}$ on the interval $[0, 2]$.
2. If $f_{\text{ave}}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a, b] = \frac{c-a}{b-a}f_{\text{ave}}[a, c] + \frac{b-c}{b-a}f_{\text{ave}}[c, b].$$

3. The linear density in a rod 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.
4. Find the exact length of the curve.

a) $y = \sqrt{1-x^2}$, $0 \leq x \leq 1$;

b) $y = \ln(\cos x)$, $0 \leq x \leq \pi/4$.

5. Find the surface area of the solid of revolution obtained by rotating the curve about the specified axis.

a) $y = \sqrt{x+4}$, $-4 \leq x \leq 2$, x -axis; b) $y = \cos x$, $0 \leq x \leq \frac{\pi}{2}$, x -axis;

c) $y = \ln x$, $1 \leq x \leq \sqrt{3}$, y -axis.

6. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a) $\int_{-\infty}^0 e^{3x} dx$;

b) $\int_0^{\infty} xe^{-2x} dx$;

c) $\int_{-\infty}^{\infty} xe^{-3x^2} dx$;

d) $\int_2^{\infty} \frac{1}{(x+3)^{3/2}} dx$;

e) $\int_2^{\infty} \frac{1}{\sqrt{x+3}} dx$;

f) $\int_{-\infty}^{\infty} \frac{x}{x^2+1} dx$;

g) $\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$;

h) $\int_1^{\infty} \frac{\ln x}{x^3} dx$;

i) $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$.

7. Use the Comparison Theorem to determine whether the integral is convergent or divergent

a) $\int_1^{\infty} \frac{\cos^2 x}{1+x^2} dx$;

b) $\int_1^{\infty} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$;

c) $\int_1^{\infty} \frac{1}{x(\sqrt{x}+1)} dx$;

d) $\int_{\pi}^{\infty} \frac{x + \sin x}{x^3} dx$;

e) $\int_1^{\infty} \frac{1}{x + e^{2x}} dx$;

f) $\int_1^{\infty} \frac{1}{\sqrt{x^3+1}} dx$.

8. Use the Limit Comparison Theorem to determine whether the integral is convergent or divergent

a) $\int_1^{\infty} \sin^2 \frac{1}{x} dx$;

b) $\int_1^{\infty} \frac{\sqrt{x}+1}{x(x+1)} dx$.